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Title: Self-Seeding, Regenerative Amplifier FEL & XFELO

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# U.S. Particle Accelerator School January 25 – February 19, 2021



## **VUV and X-ray Free-Electron Lasers**

# Self-Seeding, Regenerative Amplifier FEL & XFELO

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Time

# Monday (February 1) Lecture Outline

•	X-ray diffraction in crystals	10:00 - 10:20

- Hard X-ray self-seeding 10:20 10:50
- Break 10:50 11:00
- Soft X-ray self-seeding 11:00 11:20
- Regenerative Amplifier FEL 11:20 11:40
- XFELO 11:40 Noon



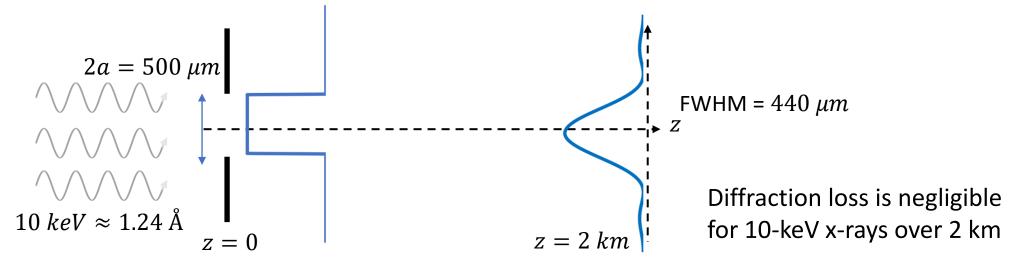


# X-ray Diffraction in Crystals





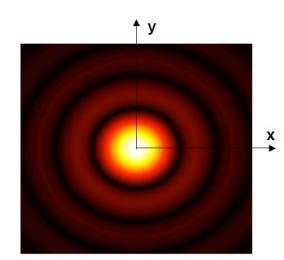
## Diffraction from an Aperture



The term diffraction is used to describe the propagation of light after passing through an aperture. The loss of power from the central cone (diffraction loss) depends on the Fresnel number, F. When F is less than 1, diffraction loss is significant.

Fresnel number

$$F = a^2/\lambda L$$



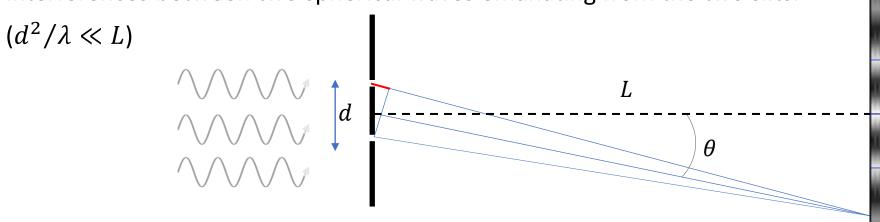
Airy disk diffraction pattern from a circular aperture





## Young Double-Slit Diffraction

Diffraction can also be the interference from the wave propagation after interacting with multiple objects of the order of a few wavelengths of the light. For example, the double-slit diffraction, which creates bright and dark fringes at a distance L, is cause by the constructive and destructive interferences between two spherical waves emanating from the two slits.



Path length difference between two waves is shown in red

Condition for constructive interference

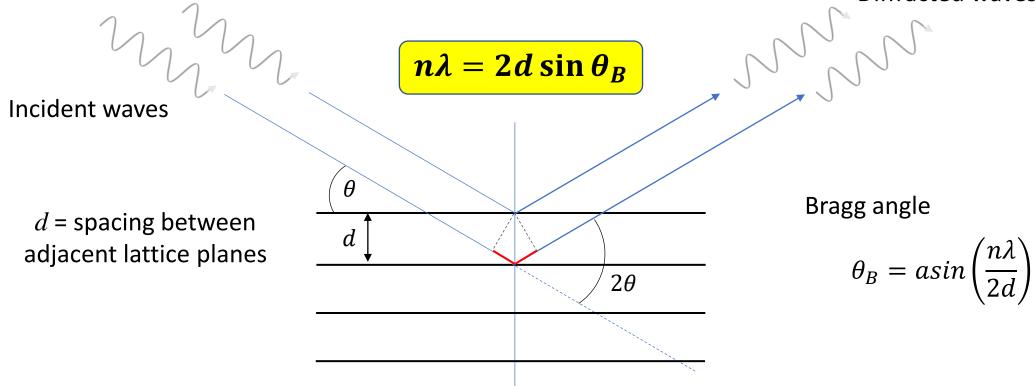
$$d \sin\theta = m\lambda$$





# **Bragg Law of Diffraction**

Diffracted waves

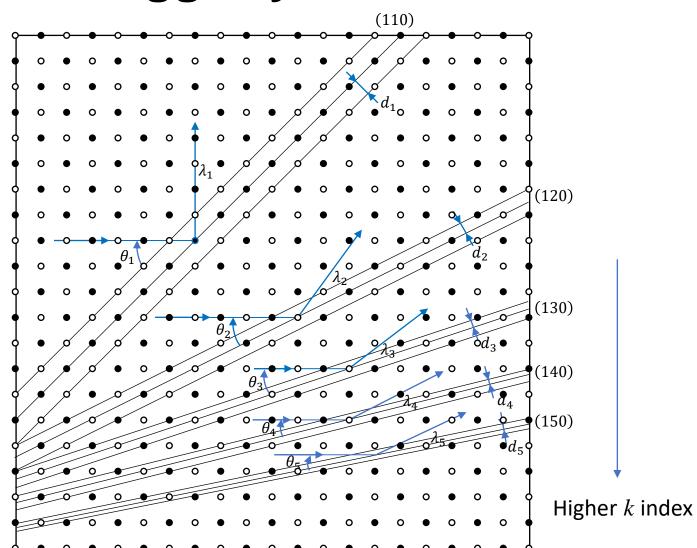


Bragg diffraction is an interference between two plane waves that are elastically scattered off lattice atoms with a spacing d between the adjacent lattice planes. The path length difference between two plane waves scattered off two adjacent lattice planes is shown in red. If the path length difference is a multiple of wavelengths, the scattered waves add constructively.





# **Bragg Crystals & Miller Indices**



d spacing for different Miller indices

$$d_{hkl} = \frac{a}{\sqrt{h^2 + k^2 + l^2}}$$

a = atomic spacings

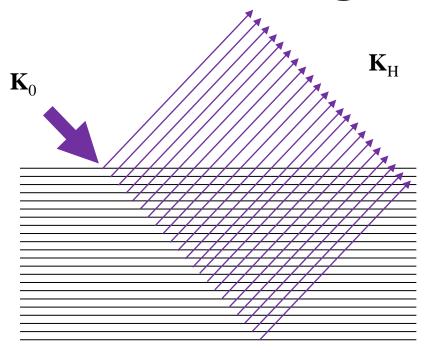
Example: Diamond a = 3.567 Å

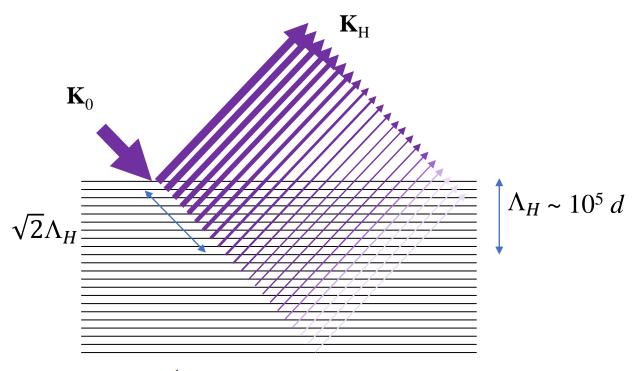
Crystal hkl	d (Å)
Diamond (111)	2.0593
Diamond (220)	1.2611
Diamond (311)	1.5704
Diamond (400)	0.8917
Diamond (331)	0.8183



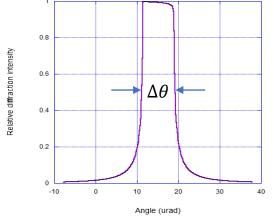


## **Extinction Length**





If the wave field penetrates an infinite distance into the crystal, all the diffracted rays would be parallel, i.e. the rocking curve would have zero width. In reality, most of the diffracted radiation is formed in a short depth in the crystal known as the extinction length. The finite "source size" gives rise to the rocking curve angular width  $\Delta\theta$ .



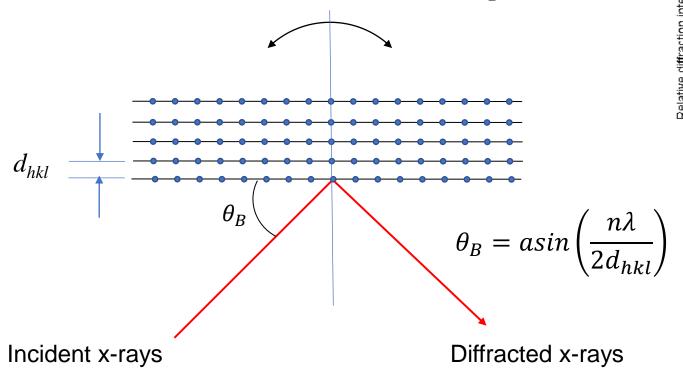
$$\Delta\theta = \frac{\lambda}{\pi\sqrt{2}\Lambda_H}$$

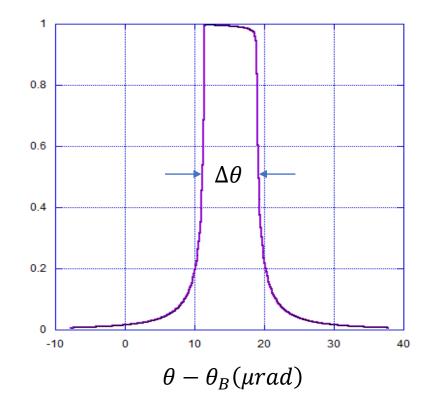




## **Rocking Curve & Darwin Width**

The term "rocking curve" comes from a method in crystallography where the crystal is tilted by a small angle off the Bragg condition to produce a plot of diffracted x-ray intensity versus offset from the Bragg angle  $\theta_B$ .



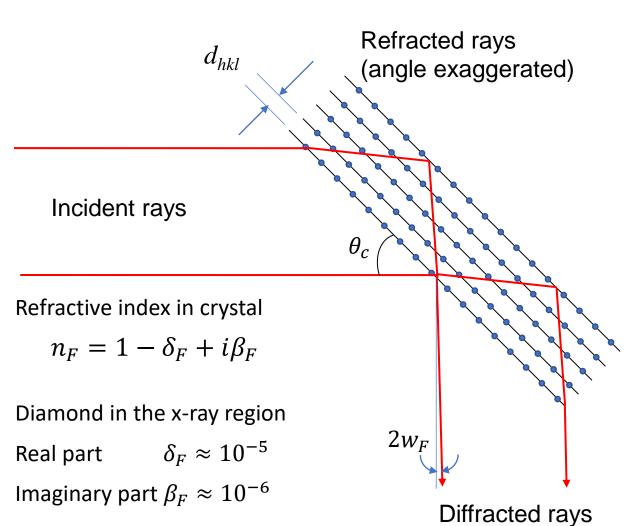


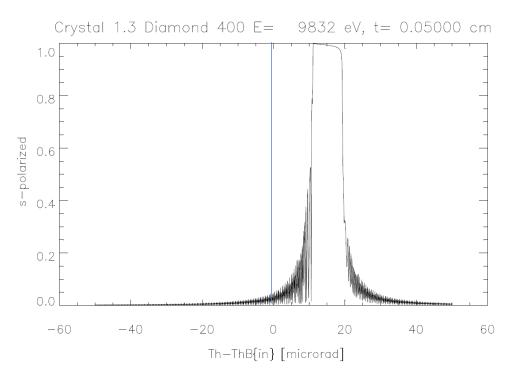
In symmetric Bragg diffraction, the incident and diffracted angles are the same as the Bragg angle  $\theta_B$ .





# **Rocking Curve Angular Shift**





Corrected expression for diffracted angle

$$2d_{hkl}\sin\theta_c = \lambda(1+w_F)$$





# **Symmetric Bragg Diffraction Crystals**

Bragg Crystal	<i>d</i> (Å)	Photon energy at 45° (keV)	Darwin Width at 45° (μrad)	Energy Width (eV)	Extinction Length (μm)	Absorption Length (μm)
Silicon (111)	3.1355	2.796	133.2	0.37	1.5	2.525
Diamond (111)	2.0593	4.257	59.4	0.25	2.2	54.79
Diamond (220)	1.2611	6.952	19.4	0.14	4.14	220.7
Diamond (311)	1.0755	8.152	8.80	0.072	7.78	591.2
Diamond (400)	0.8918	9.831	7.56	0.074	7.5	1074.2
Diamond (331)	0.8183	10.713	4.26	0.046	12.24	1413.9
Diamond (422)	0.7281	12.041	4.41	0.053	10.51	2055.5

Energy width

 $\Delta \varepsilon = \epsilon \ \Delta \theta \ \cot \theta_B$ 

Extinction length is the depth over which the atoms contribute to Bragg diffraction

Absorption length is the length over which the radiation intensity decreases to 1/e of the incident intensity due to absorption

 $\Lambda_H$  = Extinction length

 $\lambda_A$  = Absorption length



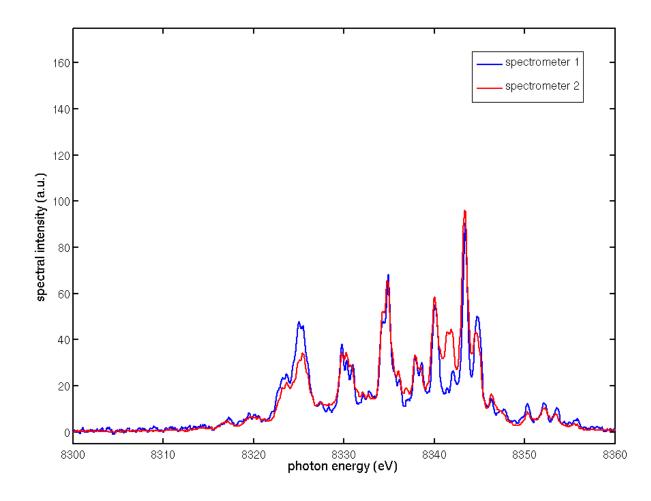


# Hard X-ray Self-Seeding





# **SASE Is Inherently Chaotic and Noisy**



Fluctuations in the spectral (energy) domain of multiple SASE pulses from the LCLS x-ray FEL





#### SASE + Monochromator = Coherent Seed



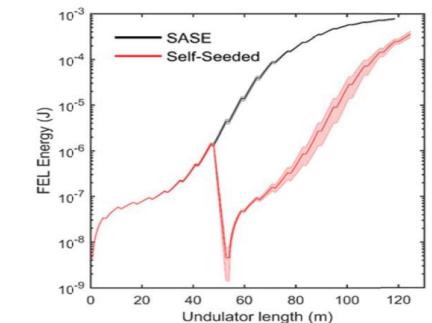
**SASE** is produced in Undulator Section 1

**Coherent seed is amplified in Undulator Section 2** 

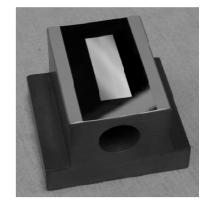
# Hard X-ray Self-Seeding (HXRSS) Monochromator = thin Bragg crystal



#### Monochromator filters SASE to produce coherent seed



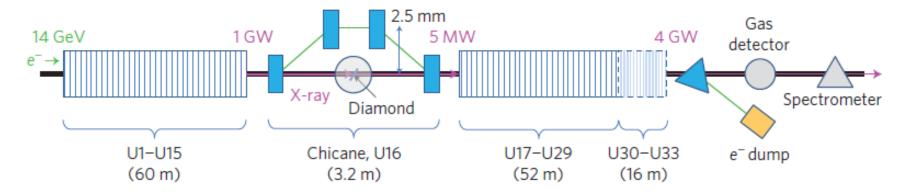
Soft X-ray Self-Seeding
Monochromator = a toroidal grating





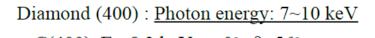


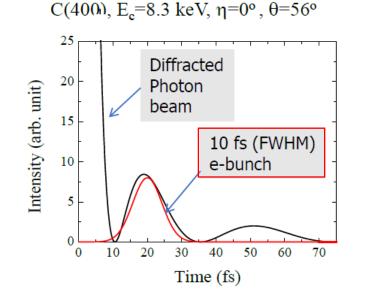
# Hard X-ray Self-Seeding Experiments

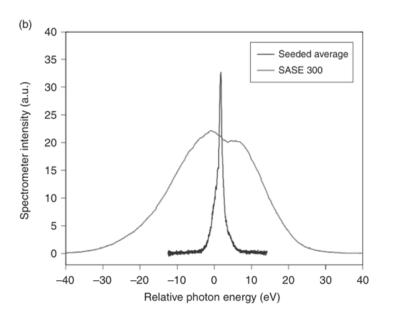


#### Chicane actions:

- wash out microbunching
- delay the electron bunch so it overlaps with one of the wake pulses
- offset the electron beam from the Bragg crystal



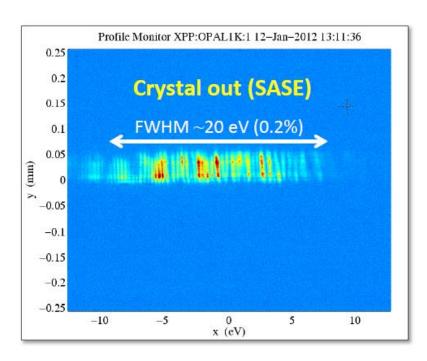


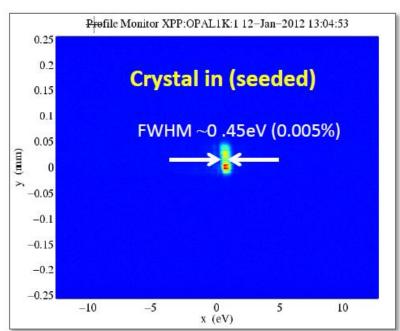


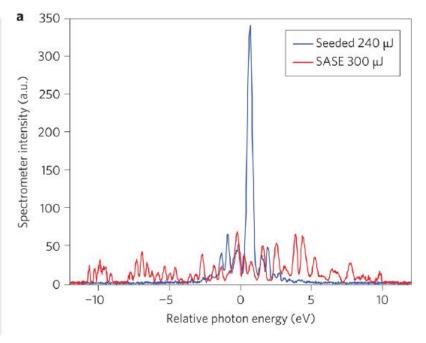




# **HXRSS Spectral Brightness Enhancement**







SASE relative BW

$$\frac{\Delta\omega}{\omega} \approx 1.5\rho \approx 2 \times 10^{-3}$$

**HXRSS** relative BW

$$\frac{\Delta\omega}{\omega} \approx 5 \times 10^{-5}$$

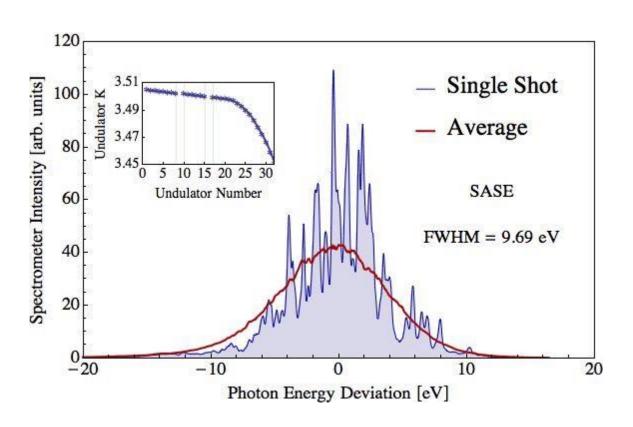
HXRSS brightness enhancement

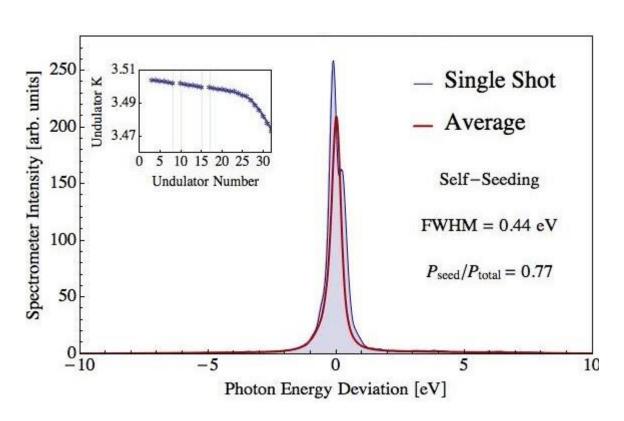
$$\frac{\mathcal{B}_{SS}}{\mathcal{B}_{SASE}} \approx 30$$





# HXRSS reduces the SASE spectral width





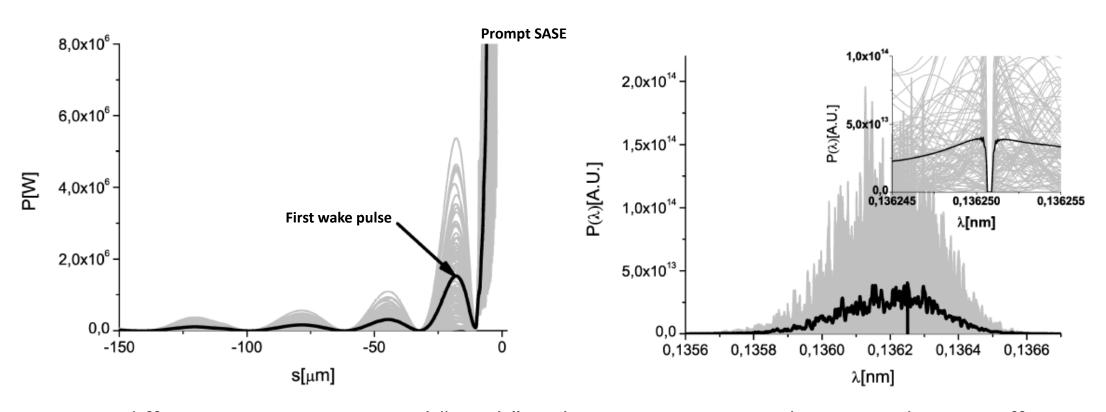
SASE spectra without self-seeding

SASE spectra with self-seeding





# How does HXRSS generate the wake pulses?

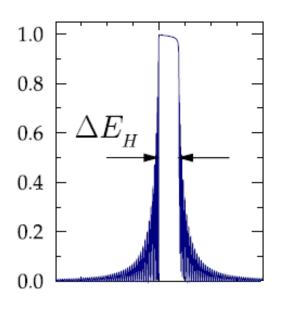


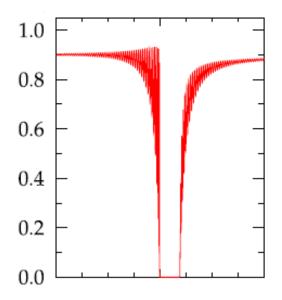
Bragg diffraction creates a spectral "notch" in the SASE spectrum. The Forward Bragg Diffraction (FBD) frequencies adjacent to the "notch" interfere constructively and destructively. In the time domain, this interference produces monochromatic wake pulses that follow the prompt SASE pulse.





## **Bragg Spectral Response Functions**





Reflected Bragg spectral response function

$$R_{0H} = R_1 R_2 \frac{1 - e^{i(\chi_1 - \chi_2)d}}{R_2 - R_1 e^{i(\chi_1 - \chi_2)d}}$$

Forward Bragg spectral response function

$$R_{00} = e^{i\chi_1 d} \frac{R_2 - R_1}{R_2 - R_1 e^{i(\chi_1 - \chi_2)d}}$$





# Reflected (Backward) Bragg Diffraction

#### **Incoming rays**

Incoming radiation profile as a function of t

$$-E_{in}(s) = E_0 F(t)$$

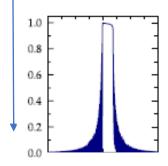
Incoming radiation spectrum as a function of  $\omega$ 

FT 
$$\mathcal{F}(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(t) e^{-i\omega t} dt$$

Bragg crystal

#### **Transmitted rays**

#### Reflected Bragg diffraction



Reflected Bragg spectrum as a function of  $\omega$ 

$$g_{0H}(\omega) = R_{0H}(\omega)\mathcal{F}(\omega)$$

Reflected Bragg radiation profile as a function of t

$$E_{0H}(t) = E_0 G_{0H}(t)$$

Inverse

FT

$$G_{0H}(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \mathcal{G}_{0H}(\omega) e^{i\omega t} d\omega$$





# **Forward Bragg Diffraction**

#### **Incoming rays**

Incoming radiation profile as a function of t

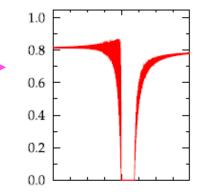
$$E_{in}(s) = E_0 F(t)$$

Incoming radiation spectrum as a function of  $\omega$ 

FT 
$$\mathcal{F}(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(t) e^{-i\omega t} dt$$

Bragg crystal

**Forward Bragg diffraction** 



Forward Bragg spectrum as a function of  $\omega$ 

$$g_{00}(\omega) = R_{00}(\omega)\mathcal{F}(\omega)$$

Forward Bragg radiation profile as a function of t

$$E_{00}(t) = E_0 G_{00}(t)$$

Inverse

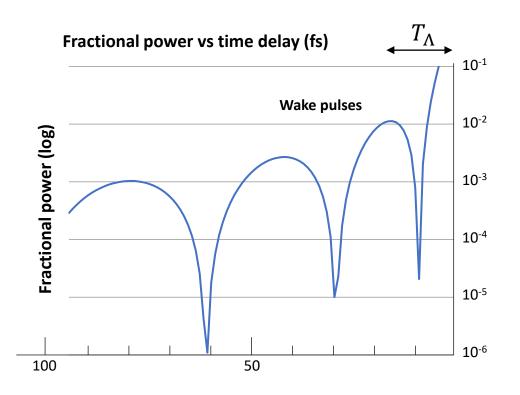
FT

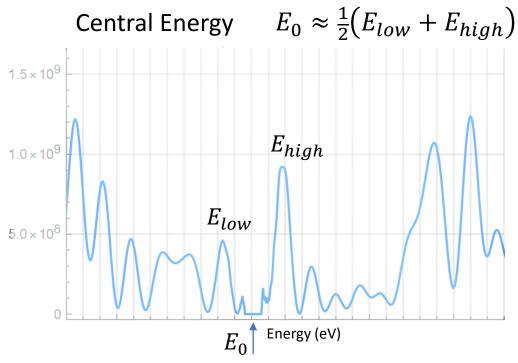
$$G_{00}(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \mathcal{G}_{00}(\omega) \ e^{i\omega t} d\omega$$





#### **Monochromatic Wake Pulses**





$$|G_{00}(t)|^2 \propto \left[ \frac{1}{2T_0} \frac{J_1\left(\sqrt{\frac{t}{T_0}}\right)}{\sqrt{\frac{t}{T_0}}} \right]^2$$

First wake pulse delay

$$T_{\Lambda} = \frac{\Lambda_H}{2\pi \ c \ sin\theta_B}$$

Characteristic time

$$T_0 = \frac{\Lambda_H^2}{2\pi^2 c d}$$

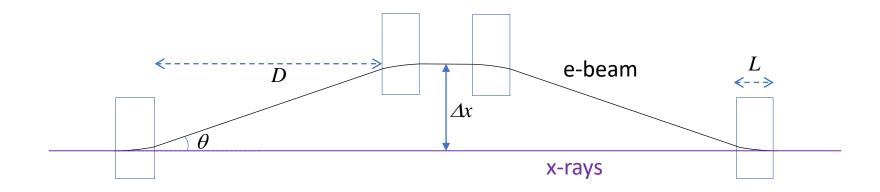
 $\Lambda_H$  = Extinction length

d =crystal thickness





## Pathlength Delay & Offset in a Chicane



Difference between the electron and x-ray beam paths

$$\Delta L = \left(\frac{4}{3}L + 2D\right) \left(\frac{1}{\cos\theta} - 1\right)$$

Electron beam delay

$$\Delta t = \frac{\Delta L}{c} \qquad c = 0.3 \ \mu m/fs$$

Chicane delay using small-angle approximation

$$\Delta L \approx \left(\frac{2}{3}L + D\right)\theta^2$$





# **Shot-to-shot Pulse Energy Fluctuations**

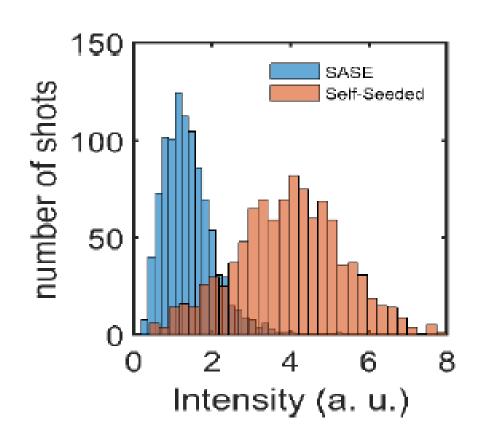
Relative pulse energy fluctuation scales with  $\sqrt{1/M}$ 

$$\frac{\sigma_W}{W} = \frac{\langle W - \langle W \rangle \rangle}{\langle W \rangle} \propto \sqrt{1/M}$$

where M = number of modes.

SASE 
$$M \approx 160$$
  $\frac{\sigma_W}{W} \approx 8\%$ 

HXRSS 
$$M \approx 12$$
  $\frac{\sigma_W}{W} \approx 30\%$ 

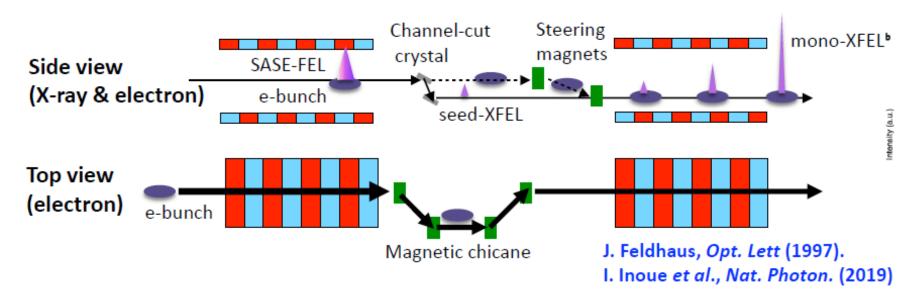


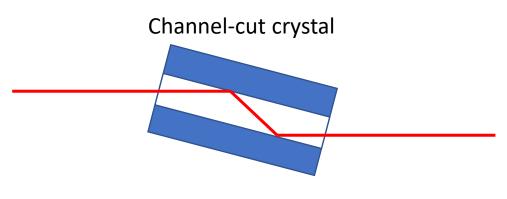
With the smaller M, HXRSS has much larger shot-to-shot pulse energy fluctuations than SASE

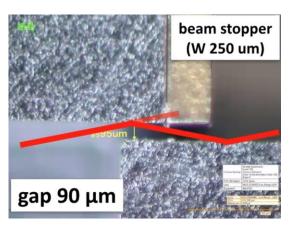


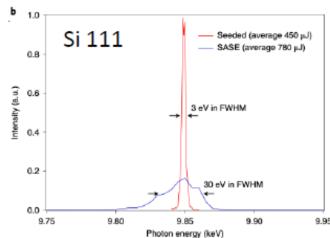


# HXR Self-Seeding with Channel-cut Crystals





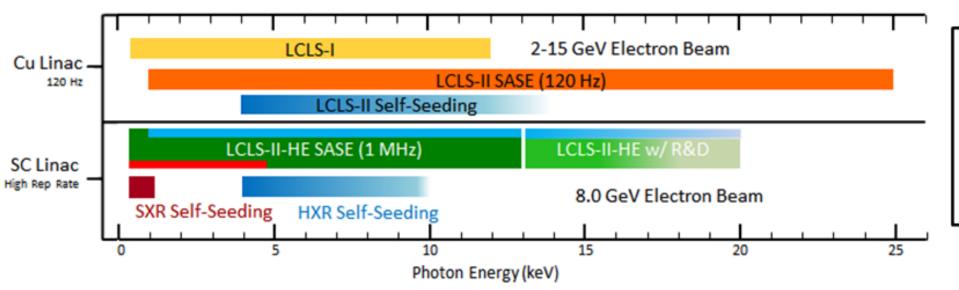


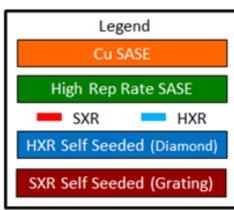






# SASE Self-Seeding Photon Energy Coverage





Gap in the Self-Seeding energy coverage between 1 and 4 keV is due to:

- Material absorption (very strong at low photon energies)
- Reduced reflectivity of gratings



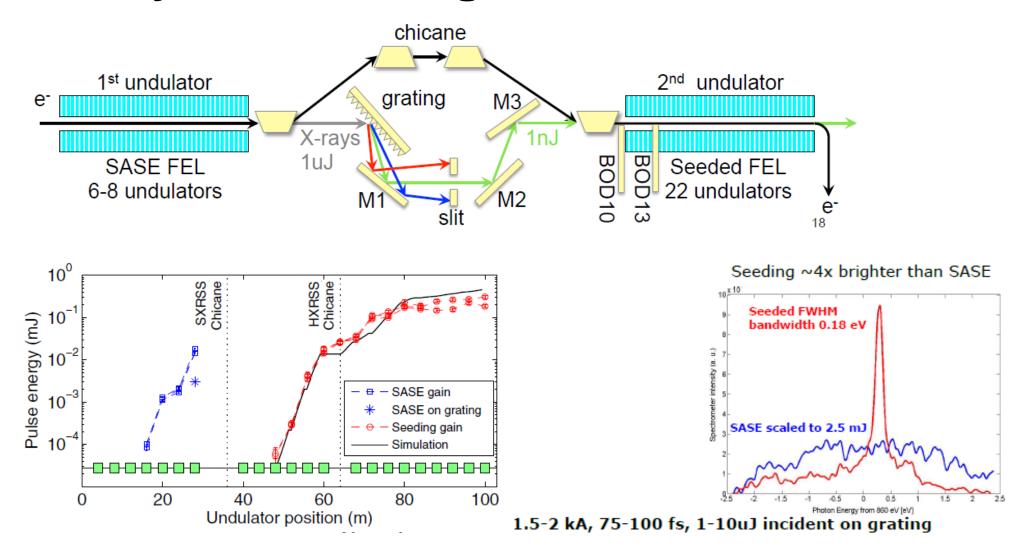


# Soft X-ray Self-Seeding





# Soft X-ray Self-Seeding at LCLS



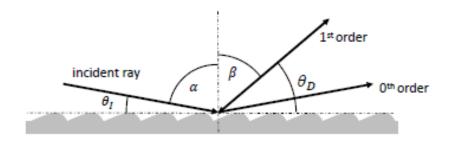




# **SXRSS Diffraction Grating**

#### **Grating equation**

$$n\lambda = d(\sin\alpha - \sin\beta)$$

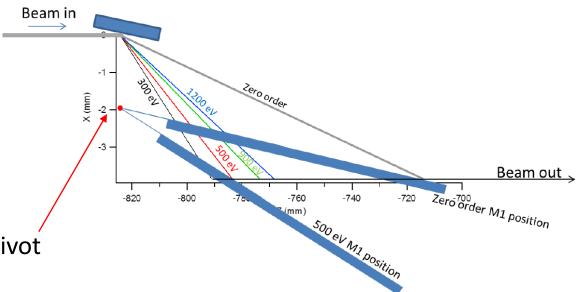


Parameter	Symbol	Value	Unit
Line spacing	d	0.4452	μ <b>m</b>
Linear coeff	$\Delta d/\Delta x$	-6.621x10 <sup>-7</sup>	
Groove height	h	15.6	nm
Grating efficiency @0.2 – 1.3 keV	$\eta_{Grating}$	3.77 - 0.58	%

For LCLS SXRSS, the incident angle lpha is fixed at 89°

The exit angle  $\beta$  depends on the x-ray energy

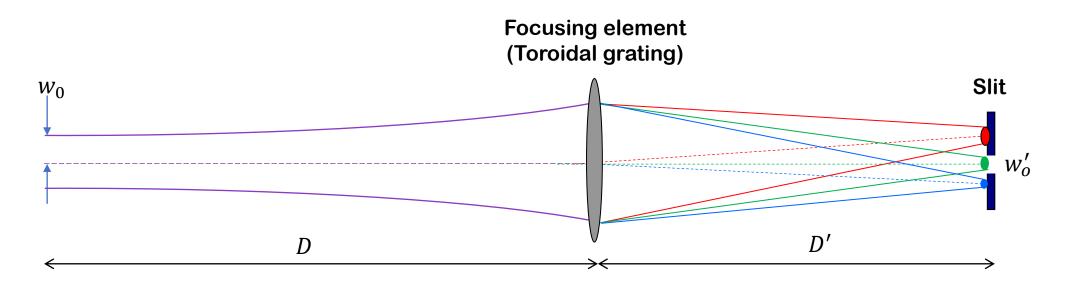
Tuning x-ray energy by rotating M1 mirror about the pivot







## Focusing Property of the Toroidal Grating



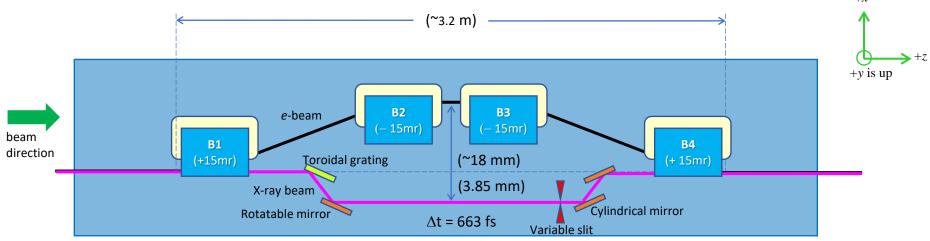
The toroidal grating focuses the x-ray beam on the slit and disperses different x-ray energies along the horizontal axis. The focusing property is energy dependent. At low x-ray energy, the SASE spectral width and  $w_o'$  are large, so the slit used to transmit the monochromatic seed with 1% bandwidth is also large. Therefore, the slit must have variable widths.

Photon energy	Vertical spot size (FWHM)	Horizontal spot size (1% SASE bandwidth)	
(eV)	(µm)	(µm)	
300	96	679	
400	88	590	
700	73	403	
1000	65	284	

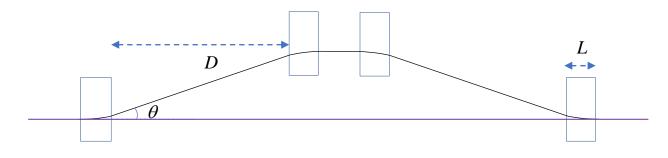




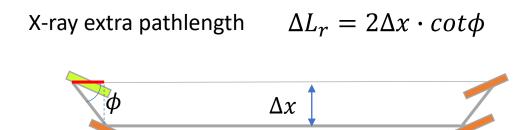
# SXRSS Delays (pathlength divided by c)



Unlike HXRSS, SXRSS must overlap the electron bunch with the main X-ray pulse



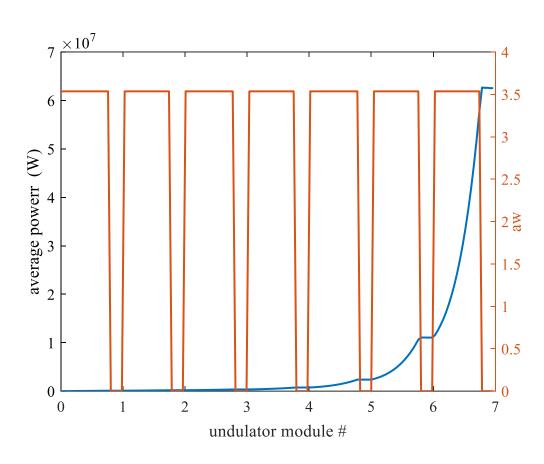
Electron extra pathlength 
$$\Delta L_b = \left(\frac{2}{3}L + D\right)\theta^2$$

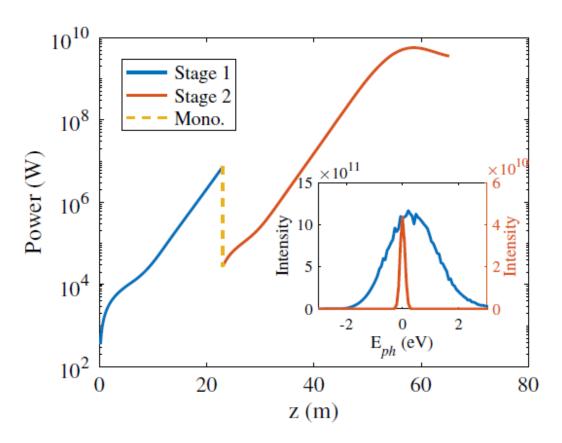






#### **SXRSS Performance at 1 keV**





Max. seed power is 20 kW (limited by grating damage)

Final SXRSS power is 8 GW





# Regenerative Amplifier FEL





# SASE, RAFEL and XFELO

	SASE	XRAFEL	XFELO
Peak Power	~10 GW	~50 GW	~100 MW
Average power	~100 W (at ~1 MHz)	10 W (at 10 kHz)	20 W (at ~1 MHz)
Spectral bandwidth	~10 eV	~0.1 eV	~1 meV
Pulse length	~ 1 − 100 fs	~ 20 fs	~ 1 ps
Stability	Poor	Excellent	Excellent
Longitudinal coherence	Poor	Excellent	Excellent
Transverse mode	Defined by gain- guiding	Defined by gain- guiding	Defined by the optical cavity





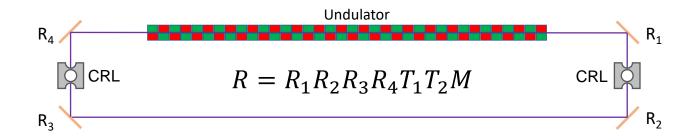
### **Characteristics of X-ray RAFEL**

- Large single-pass gain
- High reflectivity in a narrow energy band
- Saturate in a few passes
- Output pulses = a train of fs pulses separated by the cavity roundtrip time
- Output X-ray beams have both temporal and spatial coherence
- Optics re-image X-ray beam from the undulator exit to the undulator entrance
- High gain + small optical feedback = Saturation





## X-ray RAFEL with Bragg Reflectors



 $R_1R_2R_3R_4$  = Reflectivity of the Bragg reflectors within  $\Delta\theta$ 

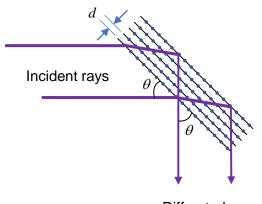
 $T_1T_2$  = Transmittivity of the two CRLs

M = Fraction of the return power that matches the FEL mode

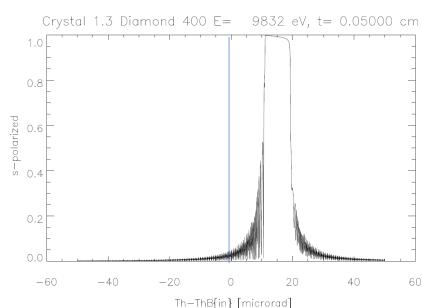
RAFEL power in the  $n^{th}$  pass

$$P_n(z) = \frac{RP_{n-1}}{9}e^{\frac{Z}{L_G}}$$

Bragg angle has to be slightly less than 45°



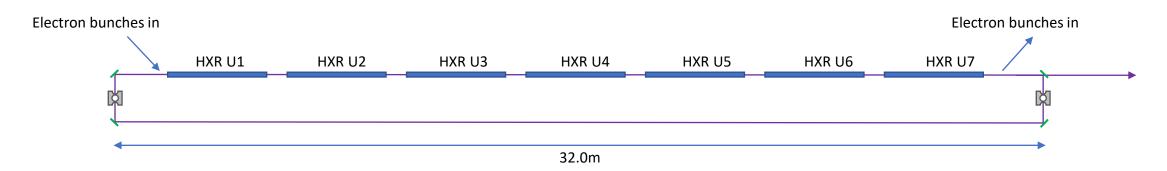
Diffracted rays

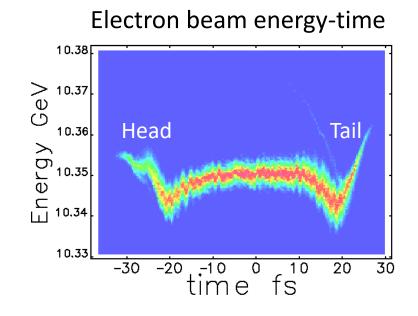




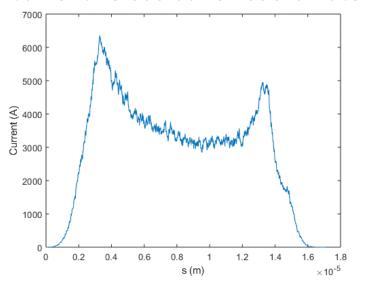


### **RAFEL Simulations with HXR Undulators**





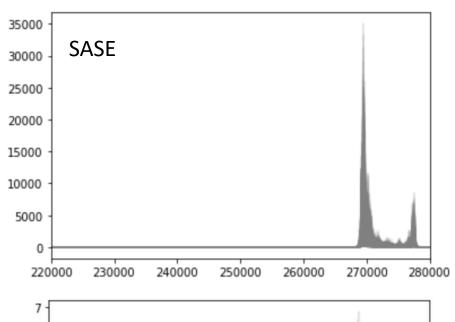
#### Current versus bunch coordinate

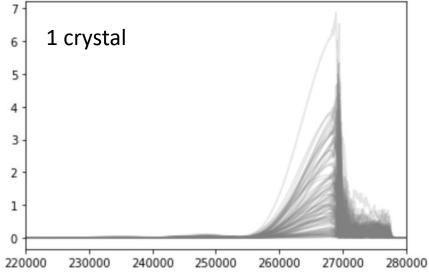


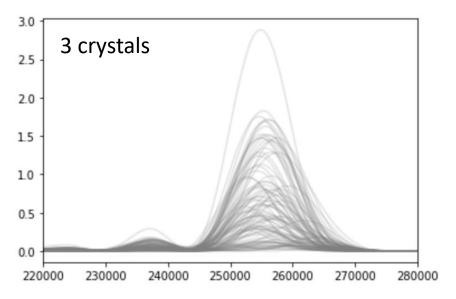


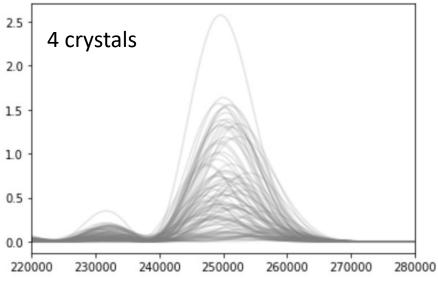


### Radiation Electric Field after 1, 3 & 4 Diffractions





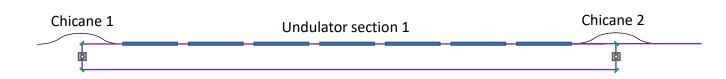


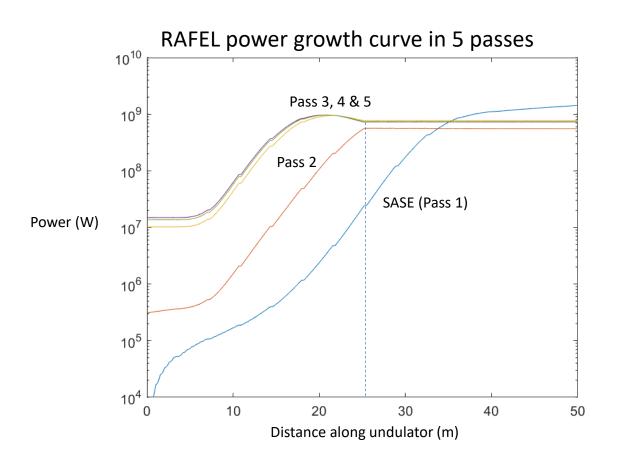


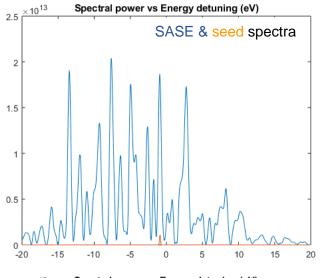


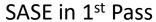


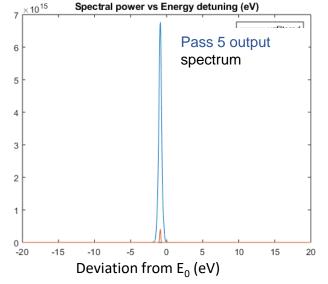
## **Expected Performance of X-ray RAFEL at LCLS**











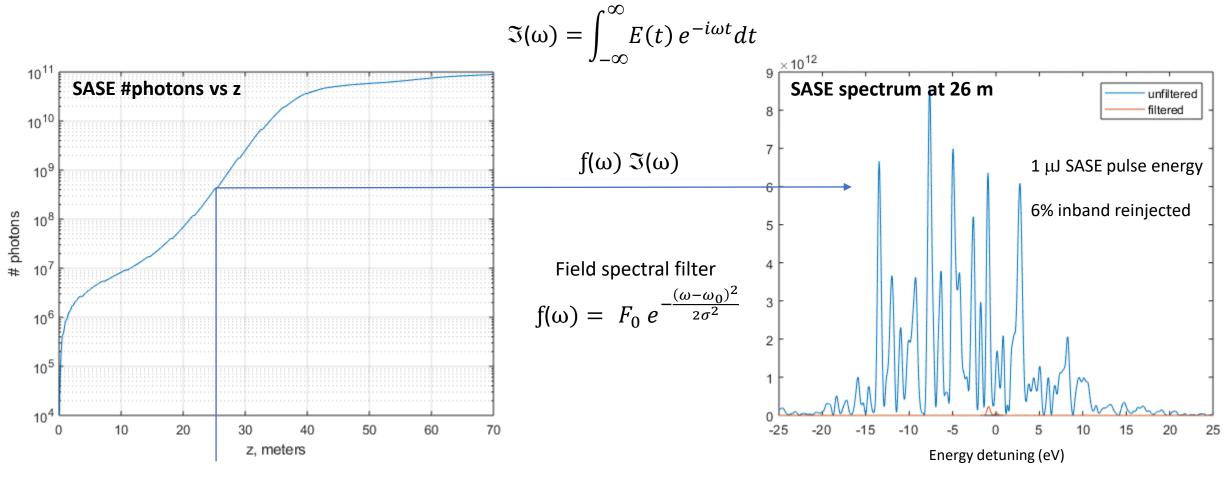
RAFEL in 5<sup>th</sup> Pass





## Pass 1 (SASE) Filtering at 9.832 keV

Calculate the field spectrum (Fourier transform of radiation field versus time)

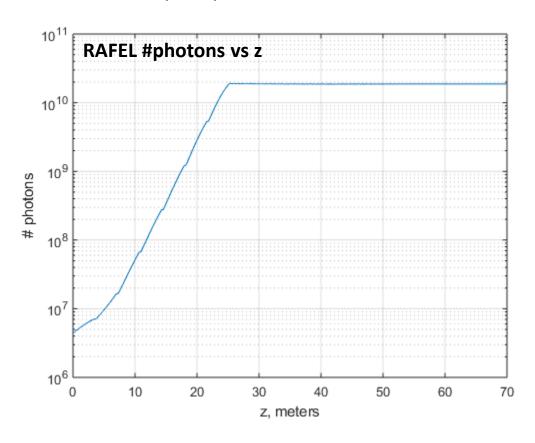


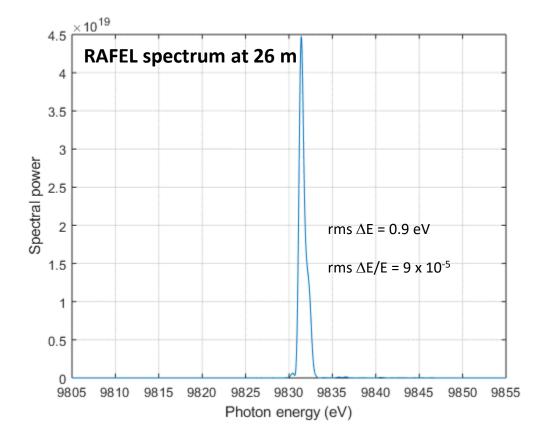




### Pass 2 Number of Photons & BW at 9.832 keV

- 6% of the filtered SASE radiation at the end of the 7<sup>th</sup> HXU is reinjected
- Power grows to >10 GW in second pass
- Output spectrum has an rms bandwidth of 0.9 eV



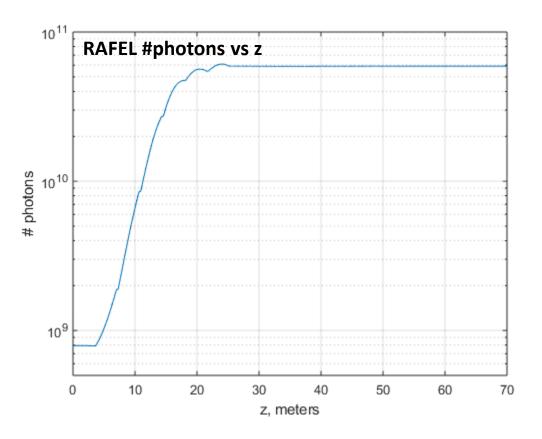


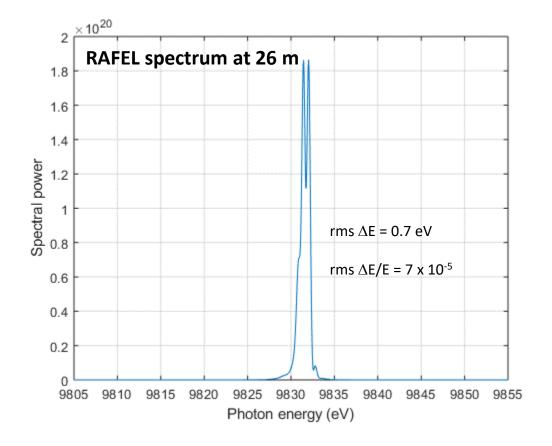




### Pass 3 Number of Photons & BW at 9.832 keV

- 6% of the coherent radiation at the end of the 7<sup>th</sup> HXU in Pass 2 is reinjected
- Power saturates at the end of the 7<sup>th</sup> undulator
- Output spectrum has an rms bandwidth of 0.7 eV.









## X-ray FEL Oscillator (XFELO)





## X-ray FEL Oscillator (XFELO)

XFELO is the X-ray version of the FEL Oscillator The first FEL (at 3.4  $\mu$ m) was an FEL Oscillator

#### First Operation of a Free-Electron Laser\*

D. A. G. Deacon,† L. R. Elias, J. M. J. Madey, G. J. Ramian, H. A. Schwettman, and T. I. Smith High Energy Physics Laboratory, Stanford University, Stanford, California 94305 (Received 17 February 1977)

A free-electron laser oscillator has been operated above threshold at a wavelength of  $3.4 \mu m$ .

Ever since the first maser experiment in 1954, physicists have sought to develop a broadly tunable source of coherent radiation. Several ingenious techniques have been developed, of which the best example is the dye laser. Most of these devices have relied upon an atomic or a molecular active medium, and the wavelength and tuning range has therefore been limited by the details of atomic structure.

Several authors have realized that the constraints associated with atomic structure would not apply to a laser based on stimulated radiation by free

electrons.<sup>1-5</sup> Our research has focused on the interaction between radiation and an electron beam in a spatially periodic transverse magnetic field. Of the schemes which have been proposed, this approach appears the best suited to the generation of coherent radiation in the infrared, the visible, and the ultraviolet, and also has the potential for yielding very high average power. We have previously described the results of a measurement of the gain at  $10.6~\mu m$ .<sup>6</sup> In this Letter we report the first operation of a free-electron laser oscillator.

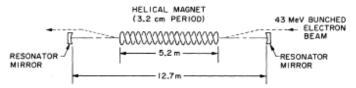
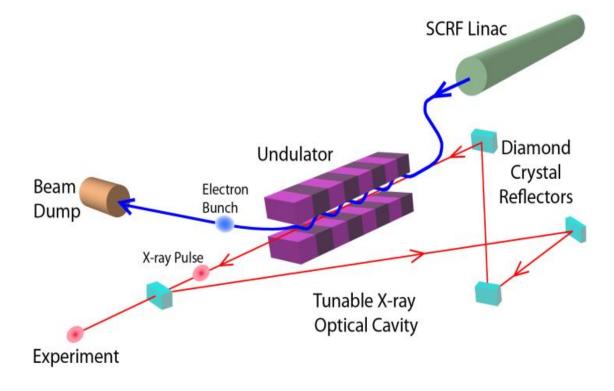


FIG. 1. Schematic diagram of the free-electron laser oscillator. (For more details see Ref. 6.)







### **FEL Oscillator Glossary**

• Small-signal gain (G<sub>SS</sub>)

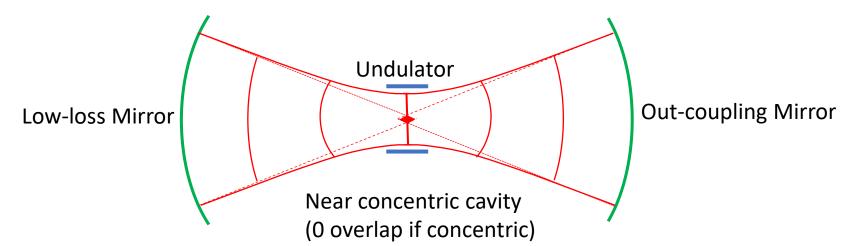
$$P_{out} = (1 + G_{ss})P_{in}$$

Saturated gain = Cavity loss

$$P_{out} = (1+L)P_{in}$$

Optical cavity

$$L_{cav} = \frac{c}{f_b}$$

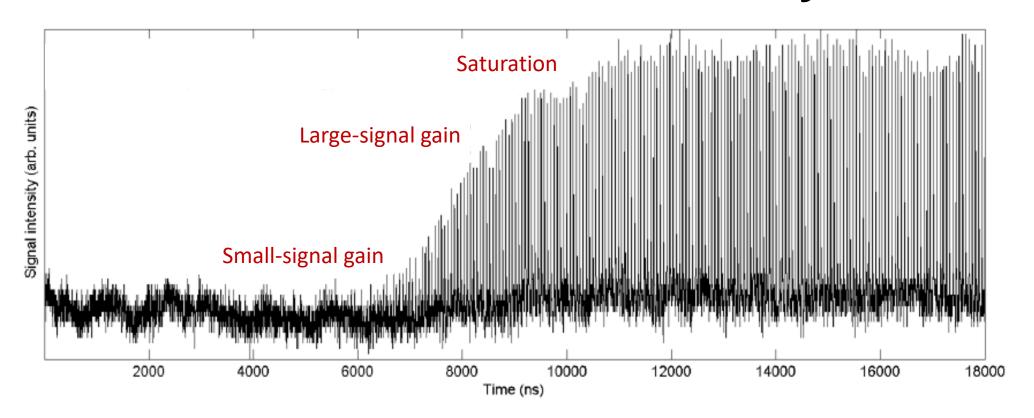


Cavity loss = Out-coupling + Mirror absorption + Diffraction





### FEL Oscillator Power Growth in Many Passes



The small-signal gain is the highest single-pass gain

Gain decreases as the optical power grows (large-signal gain)

FEL saturates when intracavity power reaches the maximum

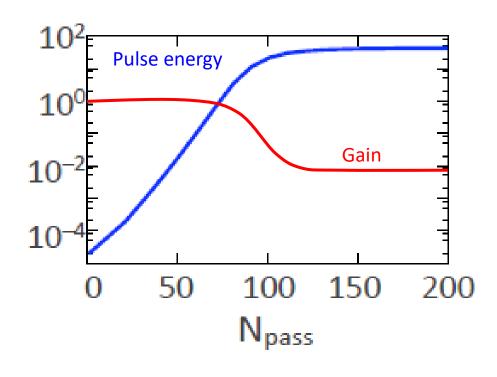
Maximum intracavity power

$$P_{in} = \frac{1}{2N_u} P_b$$





### **Power & Gain versus Number of Passes**



Start-up in an Oscillator is spontaneous emission

$$P_1 = P_s$$

Power as a function of pass number

$$P_n = R(1+G)P_{n-1} + P_s$$
  $n \ge 2$ 

$$P_n = \frac{(R(1+G))^n - 1}{R(1+G) - 1} P_s$$

*R* is the net reflectivity of all cavity mirrors

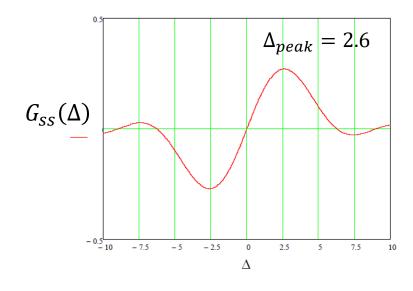
Threshold condition  $R(1 + G_{SS}) > 1$ 





### **Small-Signal Gain**

The small-signal gain peaks at a longer wavelength than the resonant wavelength (at a fixed electron beam energy) or at higher energy than the resonant energy (at a fixed wavelength).



$$G_{SS}(\Delta) = \frac{4(4\pi\rho N_u)^3}{\Delta^3} \left(1 - \cos\Delta - \frac{\Delta}{2}\sin\Delta\right)$$

Detuning 
$$\Delta = \pi N_u \eta = 2\pi N_u \frac{\Delta \lambda}{\lambda}$$

At the peak of the gain curve

$$\Delta = 2.6 \quad \Box \qquad G_{SS} = (0.0675) \ 4(4\pi\rho N_u)^3$$

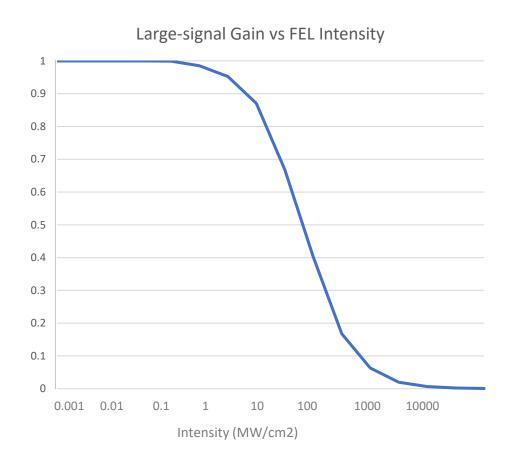
Without length constraint, the small-signal gain scales with  $N_u^3$ 

In reality, the undulator length is set at 1/3 the radiation Rayleigh range to avoid clipping the FEL beam as it diffracts and expands from the undulator center. It can be shown that  $G_{SS} \sim N_u^2$ 





### Large-Signal Gain & Saturation



Gain decreases with intracavity FEL intensity

$$G(I) = \frac{G_{SS}}{1 + \left(\frac{I}{I_{Sat}}\right)}$$

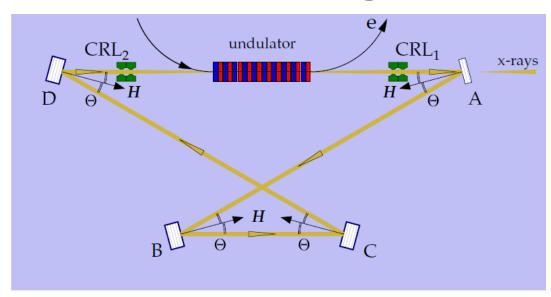
Saturated intracavity intensity

$$I_{sat}\left[\frac{MW}{cm^2}\right] \approx 100\pi \left(\frac{\gamma}{N_u}\right)^4 \frac{1}{\left(\lambda_u[cm]\widehat{K}\right)^2}$$





## **Output Coupling from XFELO Cavity**



Intracavity power

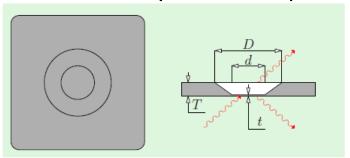
$$P_{in} = I_{sat}A_b$$

**Cavity loss** 

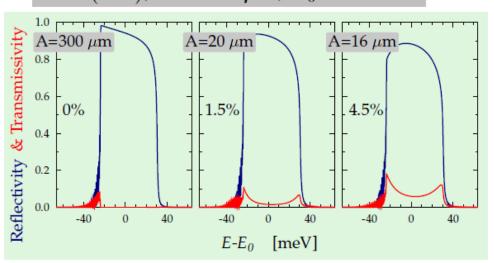
$$L = L_{out} + L_{abs} + L_{dif}$$
 Out-coupling Absorption Diffraction

 $P_o = L_{out}P_{in}$ 

#### Drumhead crystal outcoupler



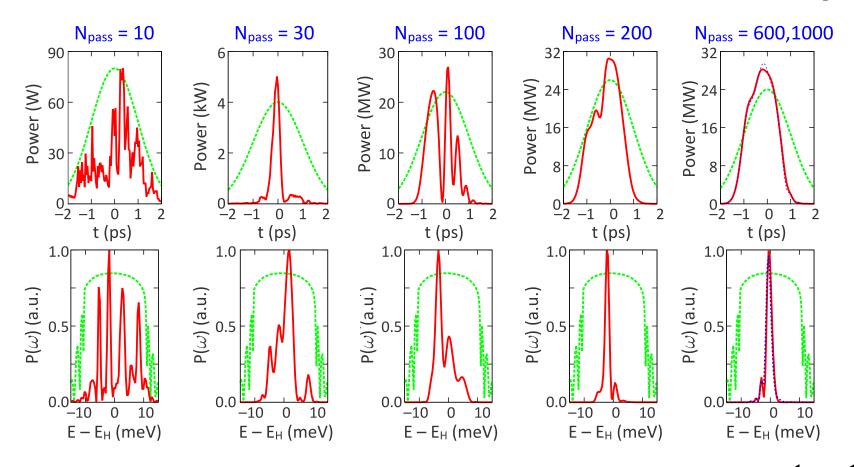
$$H=(400)$$
,  $ar{\Lambda}=3.6~\mu{
m m}$ ,  $E_0=6.9~{
m keV}$ 







# XFELO Pulse Shapes and Spectra vs. N<sub>pass</sub>



XFELO output spectra become narrower with  $N_{pass}$ 

$$\left(\frac{\sigma_E}{E}\right) \sim \frac{1}{N_u} \frac{1}{\sqrt{N_{pass}}}$$





### **Summary**

- Self-Seeding is an effective way to narrow the SASE FEL bandwidth. With fewer modes, Self-Seeding produces larger pulse-to-pulse energy fluctuations than SASE.
- Hard X-ray Self-Seeding uses the Forward Bragg Diffraction to create a notch in the SASE spectrum, resulting in coherent wake pulses in the time domain. One of these wake pulses is amplified by "fresh bunches" of electrons in subsequent undulators.
- Soft X-ray Self-Seeding uses a grating and a slit to select a narrow band of the SASE output to seed the next undulator section.
- Regenerative Amplifier FEL has been demonstrated in the IR. An on-going effort at SLAC is aimed at demonstrating RAFEL and XFELO in the hard X-ray region.
- XFELO extends the FEL Oscillator wavelengths from IR, visible and UV to the X-ray region. XFELO can potentially achieve Transform-limited bandwidth of a few meV.